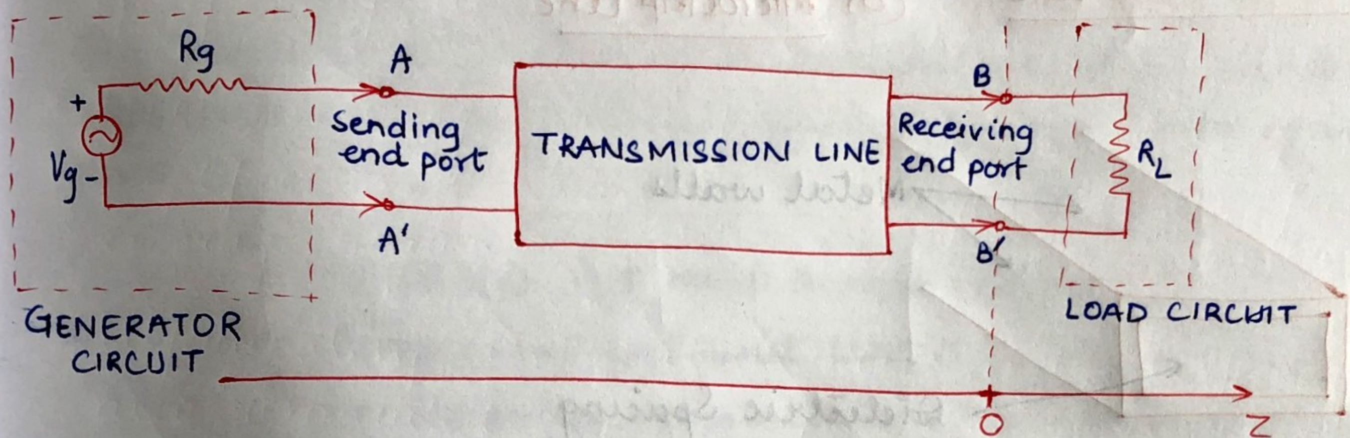
NETWORK FILTERS AND TRANSMISSION LINES (NFTL)

UNIT-4 : TRANSMISSION LINES

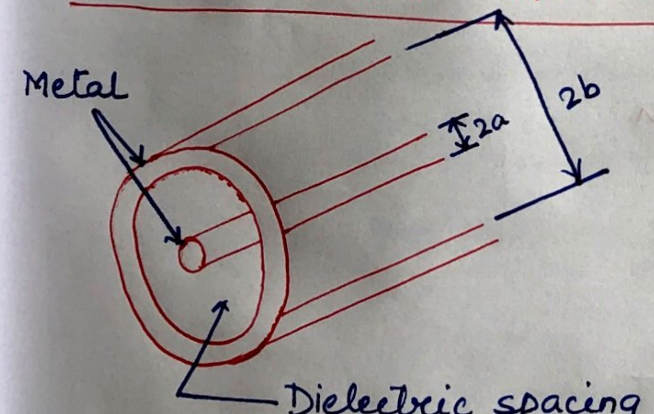
* At high frequencies, the wavelength is much smaller than the circuit size (as $f \propto \frac{1}{\lambda}$), so, the circuit theory cannot be applied. We need to use the transmission line theory.



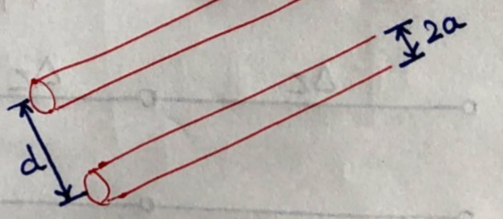
Definition: A transmission line is a two port network connecting a generator circuit at the sending end to a load at the receiving end.

* The length of a transmission line is of great importance in transmission line analysis.

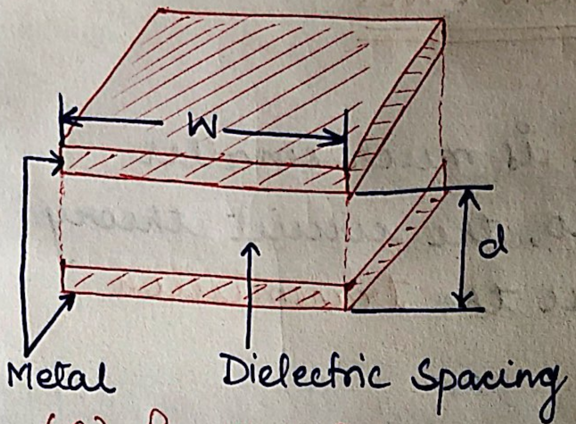
COMMON TYPES OF TRANSMISSION LINES:



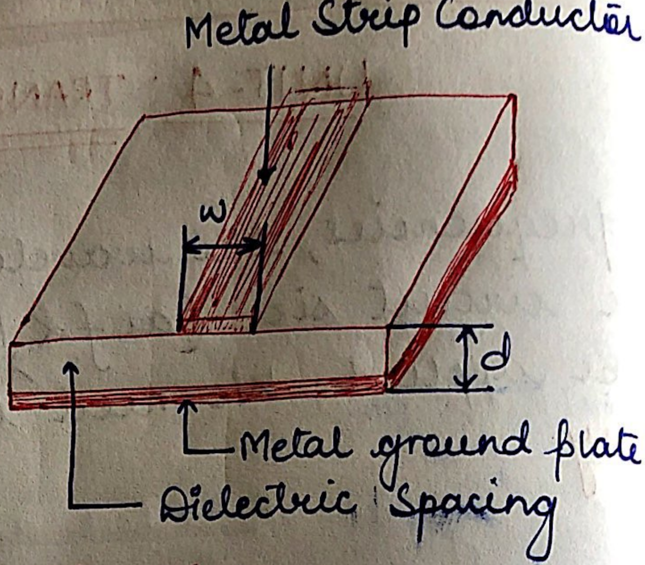
(a) Coaxial Line Cable



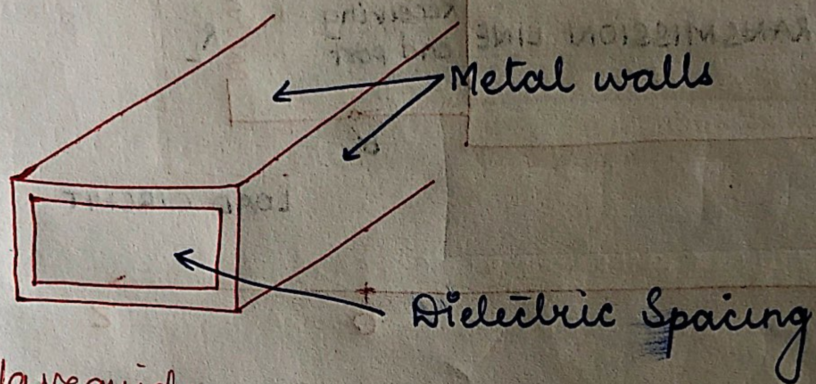
(b) Two Wire Line



(c) Parallel-Plate line



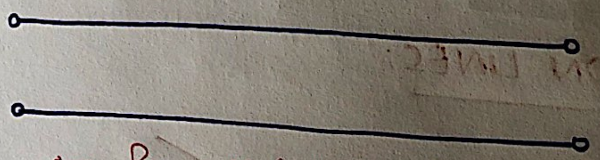
(d) Microstrip Line



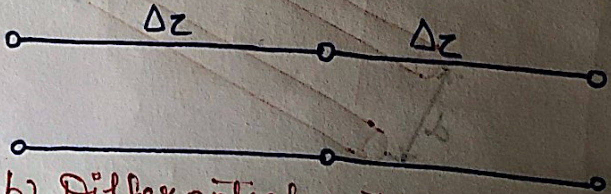
(e) Waveguide

AC STEADY STATE ANALYSIS

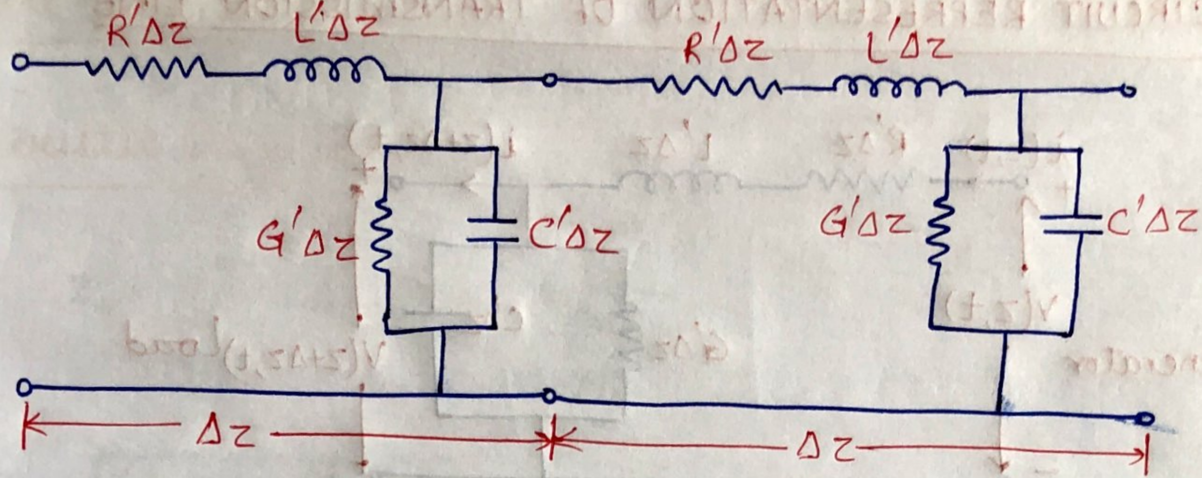
DISTRIBUTED PARAMETER REPRESENTATION



(a) Parallel Wire Representation



(b) Differential sections each Δz long



(c) Each section is represented by an equivalent circuit

The following distributed parameters are used to characterize the circuit properties of a transmission line:

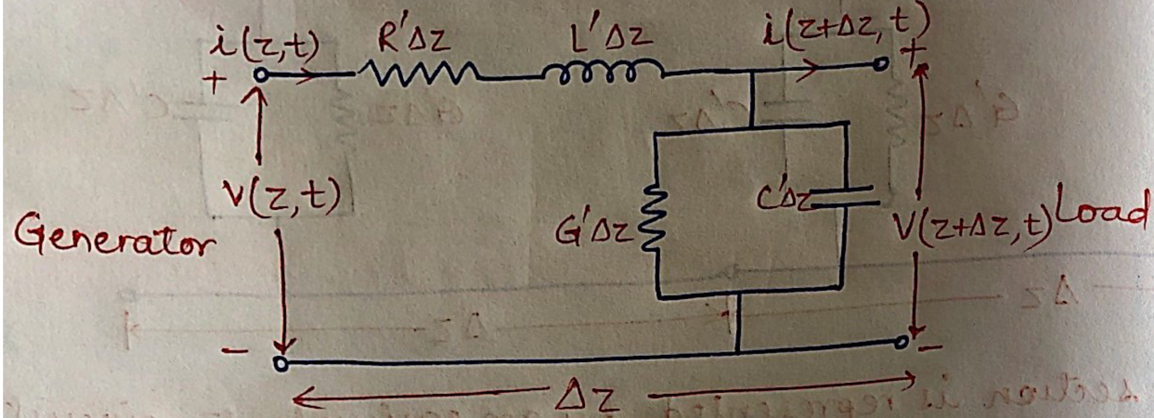
R' → Resistance per unit length (Ω/m)
L' → Inductance per unit length (H/m)
G' → Conductance per unit length ($\mho/m$)
C' → Capacitance per unit length (F/m)
Δz → Increment of length (m)

These parameters are related to the physical properties of the material filling the space between the two wires.

$$\boxed{L'C' = \mu\epsilon \text{ and } \frac{G'}{C'} = \frac{\sigma}{\epsilon}}$$

μ → permittivity
ϵ → permeability
σ → conductivity

CIRCUIT REPRESENTATION OF TRANSMISSION LINE



$$v(z,t) = \text{Real}\{V(z)e^{j\omega t}\}$$
$$i(z,t) = \text{Real}\{I(z)e^{j\omega t}\}$$

Propagation constant, $\boxed{\gamma = \alpha + j\beta = \sqrt{(R'+j\omega L')(G'+j\omega C')}}$

$\alpha \rightarrow$ Attenuation Constant (Nepers/m)

$\beta \rightarrow$ Phase Constant (rad/m)

Solution to Transmission Line Equations →

$$V(z) = V^+(z) + V^-(z)$$

$V^+(z)$ — forward travelling wave; $V^-(z)$ — Backward travelling wave.

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I(z) = I^+(z) + I^-(z)$$
$$= I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$V_0^+, V_0^-, I_0^+, I_0^- \longrightarrow$ wave amplitudes in the forward and backward directions at $z=0$

(They are complex no.'s in general)

TRANSMISSION LINE PARAMETERS

Characteristic Impedance

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \frac{R'+j\omega L'}{\gamma}$$

Since, $\gamma = \sqrt{(R'+j\omega L')(G'+j\omega C')}$

$$\Rightarrow \boxed{Z_0 = \sqrt{\frac{R'+j\omega L'}{G'+j\omega C'}}}$$

It is to be noted here that Z_0 and γ depend on distributed parameters $(R'L'G'C')$ of the line and ω but not on the length of the line.

Parameters for LOSSLESS Transmission Lines

for lossless transmission lines, $R' = G' = 0$

So, $\gamma = \sqrt{(j\omega L')(j\omega C')} = j\omega\sqrt{L'C'}$

$\Rightarrow \boxed{\alpha = 0}$ (as real part of $\gamma = 0$)

$\boxed{\beta = j\omega\sqrt{L'C'} = j\omega\sqrt{\mu\epsilon}}$ $[\because L'C' = \mu\epsilon]$

$$\boxed{V_p = \text{phase velocity} = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\mu\epsilon}}}$$

Propagation Constant, $\boxed{\gamma = j\omega\sqrt{L'C'} = j\omega\sqrt{\mu\epsilon}}$

$\because \sqrt{\mu\epsilon} = \frac{1}{V_p} = \frac{1}{f\cdot\lambda}$ $[\because v = f\lambda]$

$\therefore \gamma = j\omega \times \frac{1}{f\lambda} = j \times 2\pi f \times \frac{1}{f\lambda}$

$\therefore \boxed{\gamma = \frac{j2\pi}{\lambda}}$

Wavelength along the transmission line (λ)

$\because v_p = f \cdot \lambda \Rightarrow \lambda = \frac{v_p}{f}$

$\therefore \lambda = \frac{1}{f\sqrt{\mu\epsilon}} = \frac{\omega}{\beta} \cdot \frac{1}{f}$ $\left[\because v_p = \frac{\omega}{\beta}\right]$

$\therefore \lambda = \frac{2\pi}{\beta}$ $[\text{as } \omega = 2\pi f]$

$\therefore \lambda = \frac{2\pi}{\omega\sqrt{L'C'}} = \frac{1}{f\sqrt{L'C'}}$ $\left[\because \beta = \omega\sqrt{L'C'}\right]$

finally, $\boxed{\lambda = \frac{v_p}{f} = \frac{1}{f\sqrt{\mu\epsilon}} = \frac{\omega}{f\beta} = \frac{2\pi}{\beta} = \frac{1}{f\sqrt{L'C'}}}$

Characteristic Impedance for lossless Transmission Line:

$\because Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$

Since, for a lossless T.L., $R' = G' = 0$

Therefore, $Z_0 = \sqrt{\frac{0 + j\omega L'}{0 + j\omega C'}}$

$\Rightarrow \boxed{Z_0 = \sqrt{\frac{L'}{C'}}}$

INFINITELY LONG TRANSMISSION LINE or INFINITE LINE

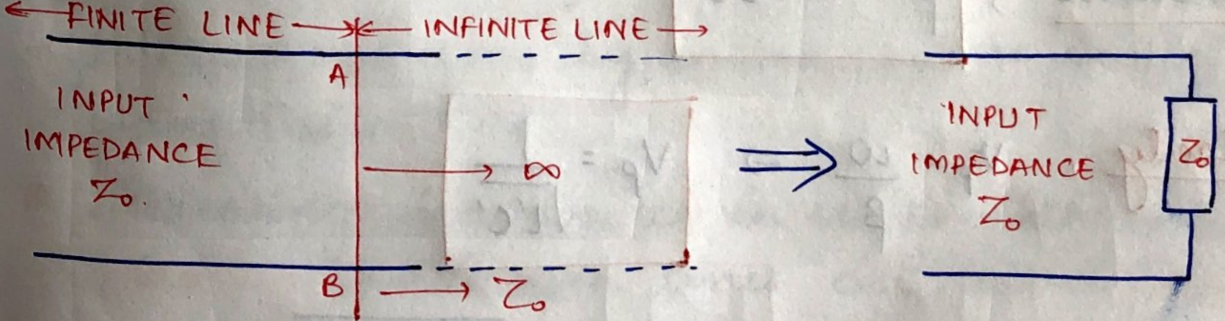
* for an infinitely long transmission line, there can be no reflected wave (backward travelling wave).
* So, for an infinitely long transmission line, there is only a forward travelling wave.

$$V(z) = V_0^+(z) \qquad [\text{as } V^-(z) = 0]$$
$$I(z) = I_0^+(z) \qquad [\text{as } I^-(z) = 0]$$

$$Z(z) = \frac{V(z)}{I(z)} = \frac{V_0^+(z)}{I_0^+(z)} = Z_0 \quad \text{——— } (1)$$

Thus, Reflection Coefficient at $z = 0$, $\boxed{\Gamma_L = 0}$

* from equation 1 above, we can say that an infinite line is equivalent to a finite line terminated in its characteristic impedance Z_0.
* If a finite ~~line~~ length of line is joined with similar kind of infinite line, their total input impedance is the same as that of infinite itself together. They make one infinite line.



CONDITION FOR MINIMUM DISTORTION OF SIGNAL ON THE LINE:

for minimum distortion of signal on the line,

$$\boxed{\frac{R'}{L'} = \frac{G'}{C'}} \quad \text{———} \quad ①$$

Therefore, $\gamma = \sqrt{(R'+j\omega L')(G'+j\omega C')}$

$$\Rightarrow \sqrt{L'\left(\frac{R'}{L'}+j\omega\right) \cdot C'\left(\frac{G'}{C'}+j\omega\right)} = \sqrt{L'C'\left(\frac{R'}{L'}+j\omega\right)\left(\frac{G'}{C'}+j\omega\right)}$$

$$\Rightarrow \gamma = \sqrt{L'C'}\left[\frac{R'}{L'}+j\omega\right] \text{ or } \sqrt{L'C'}\left[\frac{G'}{C'}+j\omega\right]$$

$$\Rightarrow \boxed{\gamma = R'\sqrt{\frac{C'}{L'}} + j\omega\sqrt{L'C'}} \text{ or } \boxed{\gamma = G'\sqrt{\frac{L'}{C'}} + j\omega\sqrt{L'C'}}$$

Attenuation Constant, $\boxed{\alpha = R'\sqrt{\frac{C'}{L'}} = G'\sqrt{\frac{L'}{C'}}}$

Phase Constant, $\boxed{\beta = \omega\sqrt{L'C'}}$

Phase velocity, $v_p = \frac{\omega}{\beta} \Rightarrow \boxed{V_p = \frac{1}{\sqrt{L'C'}}}$

Characteristic Impedance, $Z_0 = \sqrt{\frac{R'+j\omega L'}{G'+j\omega C'}}$

$$\Rightarrow Z_0 = \sqrt{\frac{L'(R'/L'+j\omega)}{C'\left(\frac{G'}{C'}+j\omega\right)}} = \sqrt{\frac{L'}{C'}} \quad \left[\because \frac{R'}{L'} = \frac{G'}{C'} \text{ from eq}^n ①\right]$$

$$\boxed{Z_0 = \sqrt{\frac{L'}{C'}}}$$

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* This shows that characteristic impedance of a distortion less line is purely real.

* In a distortion less line, all frequency components have the same attenuation (α) and phase velocity (v_p) as they are independent of ω.

* The received and transmitted waveforms have the same shape as there is no distortion in a distortion less line but the received wave is reduced in amplitude because of attenuation

* In a lossless line discussed earlier, $\alpha = 0$ and $v_p = \frac{1}{\sqrt{L'C'}}$. Both α and v_p are independent of ω. Thus, there is no distortion. So, we can say that a lossless transmission line is a distortion less transmission line too.

LOADING OF TRANSMISSION LINES

* To achieve distortion less condition in the transmission line, it is necessary to increase $\frac{L}{C}$ ratio.
* This can be done by increasing the inductance of a transmission line.
* Increasing inductance in series with line is called loading and such lines are called loaded lines.

TYPES OF LOADING

1- Lumped loading
2- Continuous loading
3- Patch loading.

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(1) Lumped Loading:

* The inductance of a transmission line can be increased by the introduction of loading coil at uniform intervals, this is called lumped loading.

* It is applicable only for a limited range of frequency.

* The loading coils have an internal resistance. Thus, increasing the total effective inductance increases resistance.

* Also, hysteresis and eddy current losses which occur in the loading coils result in increase in resistance.

* So, the loading coil should be carefully designed and placed that it does not introduce any resistance.

(2) Continuous Loading:

* A type of iron or some other magnetic material is wound on the transmission line to increase the permeability of the surrounding medium and thereby increasing the inductance.

As, $L = \frac{N^2 \mu A}{l} \Rightarrow \boxed{L \propto \mu}$

$L \rightarrow$ Inductance of coil
$N \rightarrow$ No. of turns in wire coil
$\mu \rightarrow$ Permeability of core material
$A \rightarrow$ Area of coil
$l \rightarrow$ Average length of coil.

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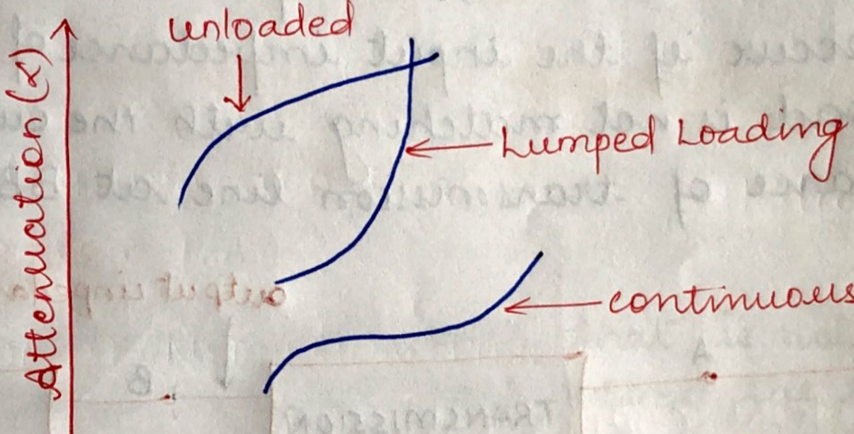
* The advantage of continuous loading over lumped loading is that attenuation factor increases uniformly with increase in frequency.

(3) Patch Loading:

* In this, there are sections of continuously loaded cable separated by sections of unloaded cable.

* The length of the section is normally a quarter kilometre

* In this method, the advantages of continuous loading is obtained and the cost is reduced considerably.



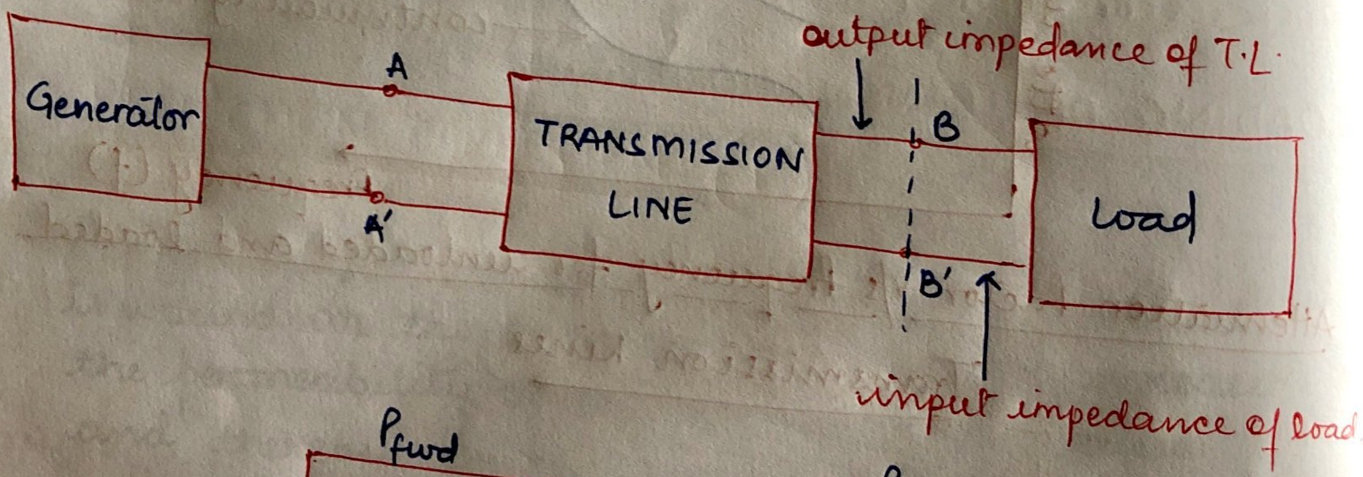
Attenuation factor v/s frequency for unloaded and loaded Transmission Lines

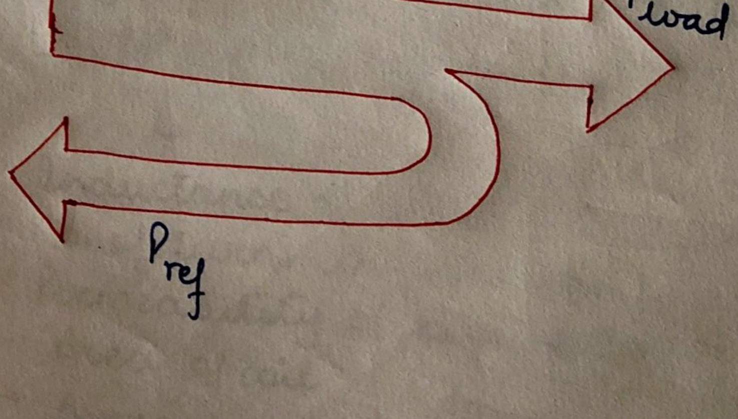
REFLECTION IN TRANSMISSION LINES

* When a wave travels on a transmission line and finds an impedance mismatch, part or all of the wave is reflected back in the opposite direction.

* Impedance mismatch occurs when the input impedance of an electrical load does not match the output impedance of the signal source.

* for example, in transmission lines, a generator is delivering power to a load through transmission line. So, an impedance mismatch will occur if the input impedance of the load is not matching with the output impedance of transmission line at BB'.





* When both forward and reflected waves travel simultaneously in opposite directions on a transmission line, the resulting wave is called a STANDING WAVE which is the superposition of the two waves.

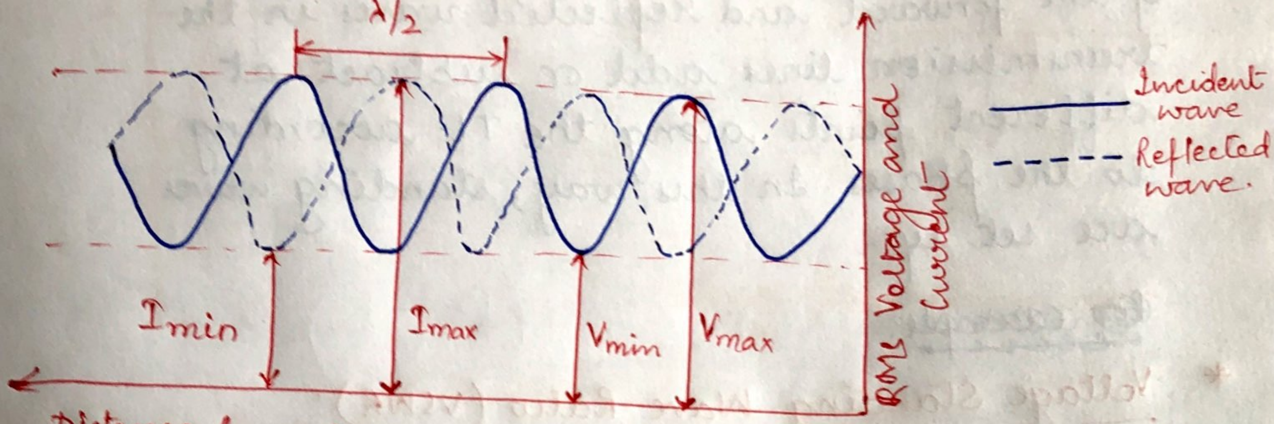


Figure for Standing Wave ~~[illegible]~~ Pattern in a Lossless Transmission Line

SWR or VSWR:

* Standing waves represent power that is not accepted by the load and reflected back along the transmission line.

* Standing Wave Ratio (SWR) or Voltage Standing Wave Ratio (VSWR) is a measurement of the level of standing waves on a transmission line.

* We know that the source (generator), transmission line and load, all have a characteristic impedance.

* In order to obtain maximum power transfer from the source to the transmission line, or the from the transmission line to the load, the impedance levels must match.

* But if there is a mismatch in impedance anywhere in the whole path, then it is not possible to transfer all the power.

* As power cannot disappear, the power that is not transferred into the load ~~to~~ travels back along the transmission line.

* When this happens, the voltages and currents of the forward and reflected waves in the transmission lines add or subtract at different points along the T.L. according to the phases. In this way, standing waves are set up.

~~For example:~~

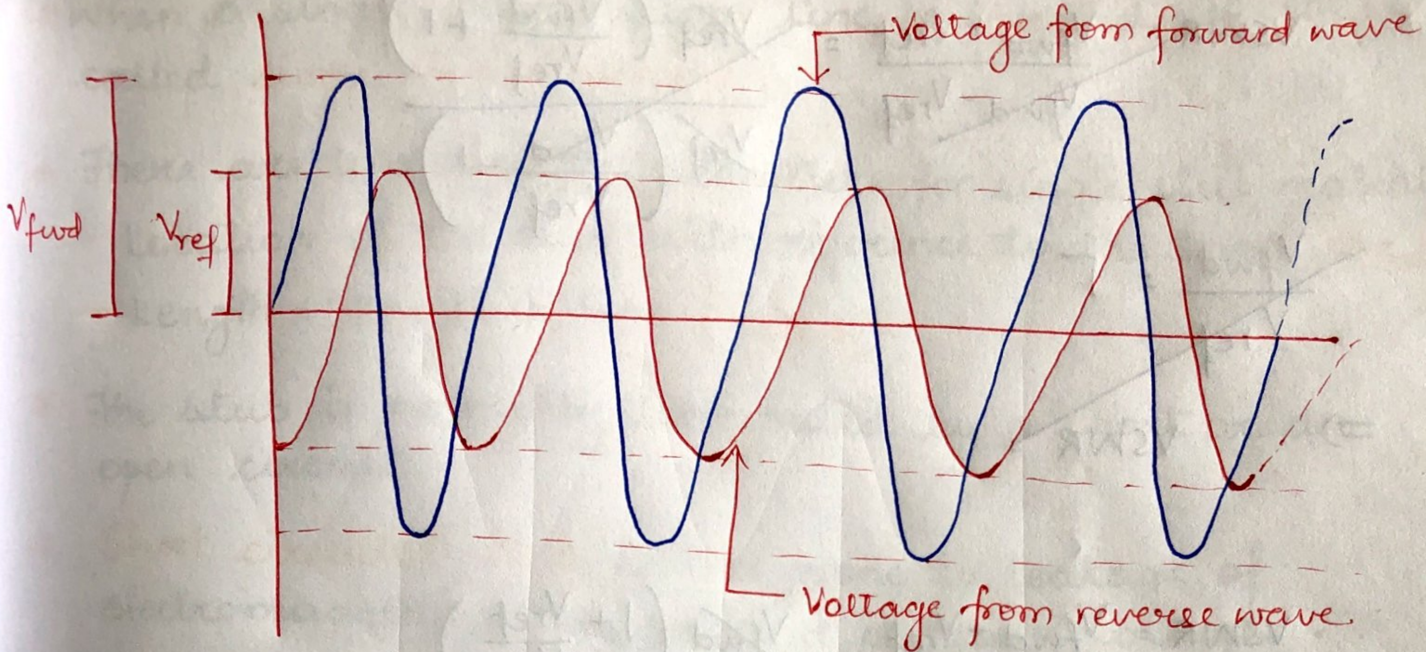
* <u>Voltage Standing Wave Ratio (VSWR)</u>:
It is defined as the ratio of the maximum to minimum voltage on a lossless line.

* A perfect impedance match has VSWR as 1:1 and a complete mismatch (like open circuit or short circuit) has VSWR as ∞:1

REFLECTION COEFFICIENT (Γ)

* It is a parameter that describes how much of an electromagnetic wave is reflected by an impedance mismatch in the transmission medium.

* It is equal to the ratio of the amplitude of the reflected wave to that of the incident wave.



Reflection Coefficient, $\boxed{\Gamma = \dfrac{V_{ref}}{V_{fwd}}}$

$V_{ref} \rightarrow$ Amplitude of reflected wave

$V_{fwd} \rightarrow$ Amplitude of incident wave

Also, $\boxed{\Gamma = \left| \dfrac{Z_L - Z_0}{Z_L + Z_0} \right|}$

$Z_L \rightarrow$ Load impedance

$Z_0 \rightarrow$ Transmission line characteristic impedance

$|\;| \rightarrow$ This is mod-sign which shows that the value of Γ is always positive.

$\Rightarrow \quad 0 \leq \Gamma \leq 1$

$\because$ Power $\propto V^2$

Thus, $\boxed{\Gamma = \sqrt{\dfrac{P_{ref}}{P_{fwd}}}}$

As discussed earlier, $\boxed{VSWR = \dfrac{V_{max}}{V_{min}}}$

Also, $\boxed{VSWR = \dfrac{V_{fwd} + V_{ref}}{V_{fwd} - V_{ref}}}$ $\Rightarrow \quad VSWR \geq 1$

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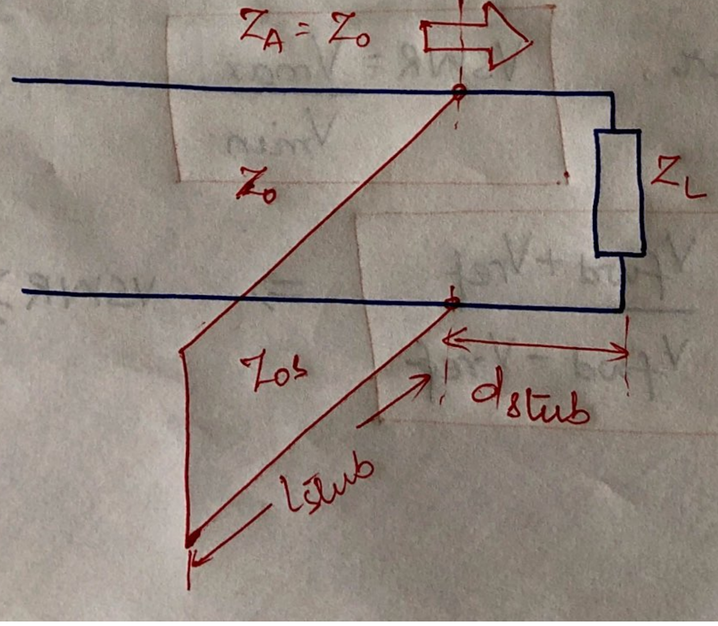
$\because$ $$VSWR = \frac{V_{fwd}+V_{ref}}{V_{fwd}-V_{ref}} = \frac{\cancel{V_{fwd}}\left(1+\frac{V_{ref}}{V_{fwd}}\right)}{\cancel{V_{fwd}}\left(1-\frac{V_{ref}}{V_{fwd}}\right)}$$

$\because$ $$\frac{V_{ref}}{V_{fwd}} = \Gamma$$

$$\Rightarrow \boxed{VSWR = \frac{1+\Gamma}{1-\Gamma}}$$

STUBS

* Impedance matching can be achieved by inserting another transmission line called stub as shown in the digram below:



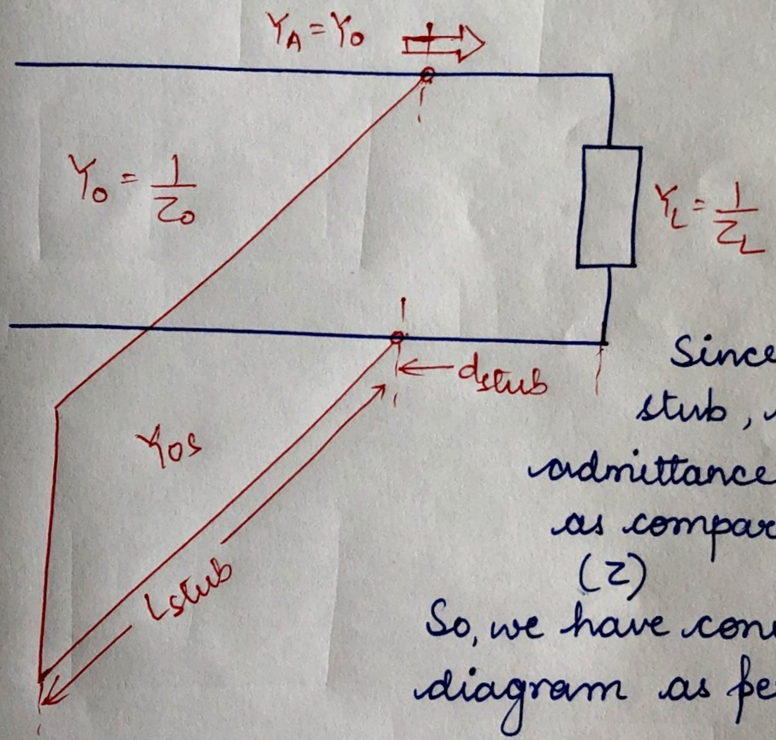
* When a single transmission line is inserted, it is called single stub matching.

* There are two design parameters for single stub matching:
  - Location of the stub with reference to the load (d_{stub})
  - Length of the stub line (L_{stub})

* The stub is normally terminated by a short or an open circuit.

* Short circuited stub is less prone to leakage of electromagnetic radiation and is easier to apply.

* Open circuited stub is more practical for certain types of transmission lines like microstrips



Since, this is a parallel stub, dealing with admittance (Y) is easier as compared to impedance (Z)

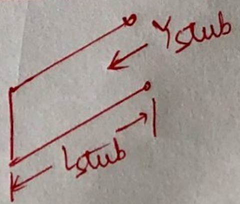
So, we have converted the diagram as per admittances.

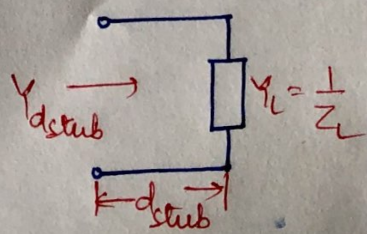
for proper impedance match,

$$\boxed{Y_A = Y_{stub} + Y_{d_{stub}} = Y_0 = \frac{1}{Z_0}}$$

Y_{stub} → Input admittance of the stub line

$Y_{d_{stub}}$ → Line admittance at location d_{stub} before stub is applied





* The drawback of this approach is that if the load is changed, the location of insertion of the stub may have to be moved.